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Fermion Tunnels of Cylindrical Symmetric Black Hole and the Corrected Entropy

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Abstract Using the method proposed by Banerjee and Majhi, we researched fermion tunneling from cylindrical symmetric black hole, and obtained the correctional entropy. In our work, we first prove that the ratios of the wave function's components are constants near the horizon, so the reasonable action form of the Dirac equation can then be gotten. From the method beyond semiclassical approximation, we finally calculate the Hawking temperature and correctional entropy.

Keywords Cylindrical symmetric black hole · Hawking radiation · Dirac equation

To prove Hawking radiation of black holes, researchers have proposed several methods [1–5]. Recently, Parikh and Wilczek have derived the quantum thermodynamics effect via tunneling method [6–36], which regards Hawking radiation as a tunneling process: the virtual particle tunnels from the inside of the horizon to the outside, where the particle will materialize becoming a real particle. Subsequently, the Hamilton-Jacobi method was put forward to research scalar particle tunneling radiation of black holes, and a fermion tunneling method was then proposed by Kerner and Mann in 2007 [37–47]. We developed prior theory and obtain the fermion's Hamilton-Jacobi method to research the higher-dimensional cases [48–50]. However, the above methods are all based on semiclassical approximation, so some small terms are left out. In 2008, Banerjee and Majhi used the method beyond semiclassical approximation to calculate all quantum corrections, and correctional Hawking temperature and entropy are finally obtained [51–61]. In this paper, we research fermion tunnel of cylindrical symmetric black hole, and the conclusion support the work of Majhi et al.

Cylindrical symmetric black hole attracts a lot of attention, because the study can help to reveal the interaction between quantum effect and spacetime geometry. In cylindrical

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symmetric space-time, the metric is given by [62–64]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2 + \alpha^2 r^2 dz^2, \tag{1}$$

where $0 \le \phi < 2\pi$, and $\alpha^2 \equiv -\Lambda/3$; r is the radial circumferential coordinate, and

$$f(r) = \alpha^2 r^2 - \frac{m}{r}, \tag{2}$$

m is related to the ADM mass density. Obviously, the horizon position r_0 satisfies the formula $f(r_0) = 0$. In this space-time, the Dirac equation with fermion mass μ is

$$\gamma^\nu D_\nu \Psi + \frac{\mu}{\hbar}\Psi = 0 \quad \nu = t, r, \phi, z, \tag{3}$$

where

$$\begin{aligned} D_\nu &= \partial_\nu + \frac{i}{2}\Gamma_\nu^{\alpha\beta}\Pi_{\alpha\beta}, \\ \Pi_{\alpha\beta} &= \frac{i}{4}\left[\gamma_\alpha, \gamma_\beta\right], \end{aligned} \tag{4}$$

In curved space-time, the gamma matrices should satisfy the relation

$$\{\gamma^\beta, \gamma^\alpha\} = 2g^{\beta\alpha} I. \tag{5}$$

Here, we can choose the gamma matrices as

$$\gamma^t = \frac{i}{\sqrt{f}}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{6}$$

$$\gamma^r = \sqrt{f}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{7}$$

$$\gamma^\phi = r^{-1}\begin{pmatrix} 0 & -i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix} = r^{-1}\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \tag{8}$$

$$\gamma^z = \frac{1}{r\alpha}\begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix} = \frac{1}{r\alpha}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

where σ^ν is the Pauli matrices. Now, let's rewritten the wave function matrix as

$$\Psi(t, r, \phi, z) = \begin{pmatrix} A(t, r, \phi, z) \\ B(t, r, \phi, z) \\ C(t, r, \phi, z) \\ D(t, r, \phi, z) \end{pmatrix}. \tag{10}$$

For the purpose of obtaining an effective Dirac equation, we should first prove that, near the horizon, the ratios between components of the wave function don't depend on t, r, ϕ and z.

To prove this conclusion, we will research the generalized metric

$$ds^2 = -f(r,t)dt^2 + f^{-1}(r,t)dr^2 + r^2 d\phi^2 + \alpha^2 r^2 dz^2. \tag{11}$$

It is clear the space-time given by (1) is a special case of (11), so the above conclusion is true in static space-time if it is true in space-time (11). In fact, when we consider the evaporation effect of black hole, the mass of black hole should depend on the time [65]. In the space-time (11), the Dirac equation is

$$\sqrt{f}\left(\frac{\partial C}{\partial r} - \frac{f'C}{4f} - \frac{C}{r}\right) + \frac{\mu}{\hbar}A + \frac{i}{\sqrt{f}}\left(\frac{\partial A}{\partial t} + \frac{\dot{f}A}{4f}\right) - \frac{i}{r}\frac{\partial C}{\partial \phi} - \frac{i}{\alpha r}\frac{\partial D}{\partial z} = 0, \tag{12}$$

$$\sqrt{f}\left(\frac{\partial D}{\partial r} - \frac{f'D}{4f} - \frac{D}{r}\right) + \frac{\mu}{\hbar}B + \frac{i}{\sqrt{f}}\left(\frac{\partial B}{\partial t} + \frac{\dot{f}B}{4f}\right) + \frac{i}{r}\frac{\partial D}{\partial \phi} - \frac{i}{\alpha r}\frac{\partial C}{\partial z} = 0, \tag{13}$$

$$\sqrt{f}\left(\frac{\partial A}{\partial r} - \frac{f'A}{4f} - \frac{A}{r}\right) + \frac{\mu}{\hbar}C - \frac{i}{\sqrt{f}}\left(\frac{\partial C}{\partial t} + \frac{\dot{f}C}{4f}\right) + \frac{i}{r}\frac{\partial A}{\partial \phi} + \frac{i}{\alpha r}\frac{\partial B}{\partial z} = 0, \tag{14}$$

$$\sqrt{f}\left(\frac{\partial B}{\partial r} - \frac{f'B}{4f} - \frac{B}{r}\right) + \frac{\mu}{\hbar}D - \frac{i}{\sqrt{f}}\left(\frac{\partial D}{\partial t} + \frac{\dot{f}D}{4f}\right) - \frac{i}{r}\frac{\partial B}{\partial \phi} + \frac{i}{\alpha r}\frac{\partial A}{\partial z} = 0. \tag{15}$$

where $\dot{f} = \frac{\partial f}{\partial t}$, and $f' = \frac{\partial f}{\partial r}$. From (12) and (14), we can obtain

$$\frac{\frac{\partial C}{\partial r} - \frac{f'C}{4f} - \frac{C}{r}}{\frac{\partial A}{\partial r} - \frac{f'A}{4f} - \frac{A}{r}} = \frac{i(\frac{\partial A}{\partial t} + \frac{\dot{f}A}{4f}) - \sqrt{f}(\frac{i}{r}\frac{\partial C}{\partial \phi} + \frac{i}{r\alpha}\frac{\partial D}{\partial z} - \frac{\mu}{\hbar}A)}{-i(\frac{\partial C}{\partial t} + \frac{\dot{f}C}{4f}) + \sqrt{f}(\frac{i}{r}\frac{\partial A}{\partial \phi} + \frac{i}{r\alpha}\frac{\partial B}{\partial z} - \frac{\mu}{\hbar}C)}. \tag{16}$$

Near the horizon, due to $f \to 0$, the formula is given by

$$\begin{aligned}&\left(\frac{\partial C}{\partial r}\frac{\partial C}{\partial t} + \frac{\partial A}{\partial r}\frac{\partial A}{\partial t}\right) - \left(\frac{f'}{4f} + \frac{1}{r}\right)\frac{\dot{f}}{4f}(C^2 + A^2) - \left(\frac{f'}{4f} + \frac{1}{r}\right)\left(C\frac{\partial C}{\partial t} + A\frac{\partial A}{\partial t}\right)\\ &\quad + \frac{\dot{f}}{4f}\left(C\frac{\partial C}{\partial r} + A\frac{\partial A}{\partial r}\right) = 0.\end{aligned} \tag{17}$$

Because f depend on the position of horizon r_0 and time t, from the equation above, we can obtain the formulas

$$\begin{gathered}\frac{\partial C}{\partial r}\frac{\partial C}{\partial t} + \frac{\partial A}{\partial r}\frac{\partial A}{\partial t} = 0,\\ C^2 + A^2 = 0,\\ C\frac{\partial C}{\partial t} + A\frac{\partial A}{\partial t} = 0,\\ C\frac{\partial C}{\partial r} + A\frac{\partial A}{\partial r} = 0.\end{gathered} \tag{18}$$

From (18), the relation between A and C is determined

$$C^2 + A^2 = 0. \tag{19}$$

Similarly, near the horizon, we can also prove that

$$B^2 + D^2 = 0. \tag{20}$$

and our conclusion is in accordance with the results in Refs. [37, 38, 55, 62, 63]. Using the above method, from (12), (14) and (19), (20), we can get

$$\frac{\partial}{\partial t}\left(\frac{A}{B}\right) = \frac{\partial}{\partial r}\left(\frac{A}{B}\right) = \frac{\partial}{\partial t}\left(\frac{C}{D}\right) = \frac{\partial}{\partial r}\left(\frac{C}{D}\right) = 0. \tag{21}$$

In static space-time, (19), (20) and (21) are true all the same, because space-time which is given by (1) is a special case of (11). Via these results, we now rewrite the Dirac equation as an action form. In static space-time, we can rewrite A as

$$A(t, r, \phi, z) = E e^{\frac{i}{\hbar}(-\omega t + R(r))} Y(\phi, z), \tag{22}$$

so C can also be rewritten as

$$C(t, r, \phi, z) = G e^{\frac{i}{\hbar}(-\omega t + R(r))} Y(\phi, z), \tag{23}$$

and B and D can be rewritten as

$$B(t, r, \phi, z) = F e^{\frac{i}{\hbar}(-\omega t + R(r))} \hat{Y}(\phi, z), \tag{24}$$

$$D(t, r, \phi, z) = H e^{\frac{i}{\hbar}(-\omega t + R(r))} \hat{Y}(\phi, z). \tag{25}$$

Observably, the relation between constants E and G is $E = \pm iG$, and the relation between constants G and H is $G = \pm iH$. It is not necessary to discuss the relation between $Y(\phi, z)$ and $\hat{Y}(\phi, z)$, because tunneling near the horizon is radial, and only the $(r - t)$ sector is important. Therefore, the action form's Dirac equation can be given by

$$\frac{\omega E}{\sqrt{f}} + \sqrt{f}\left(i\frac{\partial R}{\partial r} - \frac{\hbar f'}{4f} - \frac{\hbar}{r}\right)G + \frac{\lambda_1 G}{r} + \mu E = 0, \tag{26}$$

$$\frac{\omega F}{\sqrt{f}} + \sqrt{f}\left(i\frac{\partial R}{\partial r} - \frac{\hbar f'}{4f} - \frac{\hbar}{r}\right)H + \frac{\lambda_2 H}{r} + \mu F = 0, \tag{27}$$

$$-\frac{\omega G}{\sqrt{f}} + \sqrt{f}\left(i\frac{\partial R}{\partial r} - \frac{\hbar f'}{4f} - \frac{\hbar}{r}\right)E - \frac{\lambda_1 E}{r} + \mu G = 0, \tag{28}$$

$$-\frac{\omega H}{\sqrt{f}} + \sqrt{f}\left(i\frac{\partial R}{\partial r} - \frac{\hbar f'}{4f} - \frac{\hbar}{r}\right)F - \frac{\lambda_2 F}{r} + \mu H = 0. \tag{29}$$

In WKB approximation, we can decompose radial action and energy of 1/2 spin particle as

$$R(r) = \sum_{i=0}^{\infty} \hbar^i R_i(r), \tag{30}$$

$$\omega = \sum_{i=0}^{\infty} \hbar^i \omega, \tag{31}$$

where R_0 and ω_0 is semiclassical radial action and energy of fermion. Substituting (30) and (31) into the Dirac equation and then equating the different powers of $\hbar$ on both sides, we can get following equations

$$\begin{aligned}
\hbar^0:\quad & \frac{\omega_0 E}{\sqrt{f}} + i\sqrt{f}\frac{dR_0}{dr}G + \frac{\lambda_1 G}{r} + \mu E = 0,\\
& \frac{\omega_0 F}{\sqrt{f}} + i\sqrt{f}\frac{dR_0}{dr}H + \frac{\lambda_2 H}{r} + \mu F = 0,\\
& -\frac{\omega_0 G}{\sqrt{f}} + i\sqrt{f}\frac{dR_0}{dr}E - \frac{\lambda_1 E}{r} + \mu G = 0,\\
& -\frac{\omega_0 H}{\sqrt{f}} + i\sqrt{f}\frac{dR_0}{dr}F - \frac{\lambda_2 F}{r} + \mu H = 0,
\end{aligned} \tag{32}$$

$$\begin{aligned}
\hbar^1:\quad & \frac{\omega_1 E}{\sqrt{f}} + \sqrt{f}\left(i\frac{dR_1}{dr} - \frac{f'}{4f} - \frac{1}{r}\right)G = 0,\\
& \frac{\omega_1 F}{\sqrt{f}} + \sqrt{f}\left(i\frac{dR_1}{dr} - \frac{f'}{4f} - \frac{1}{r}\right)H = 0,\\
& -\frac{\omega_1 G}{\sqrt{f}} + \sqrt{f}\left(i\frac{dR_1}{dr} - \frac{f'}{4f} - \frac{1}{r}\right)E = 0,\\
& -\frac{\omega_1 H}{\sqrt{f}} + \sqrt{f}\left(i\frac{dR_1}{dr} - \frac{f'}{4f} - \frac{1}{r}\right)F = 0,
\end{aligned} \tag{33}$$

$$\begin{aligned}
\hbar^k:\quad & k \ge 2 \quad \frac{\omega_k E}{\sqrt{f}} + i\sqrt{f}\frac{dR_k}{dr}G = 0,\\
& \frac{\omega_k F}{\sqrt{f}} + i\sqrt{f}\frac{dR_k}{dr}H = 0,\\
& -\frac{\omega_k G}{\sqrt{f}} + i\sqrt{f}\frac{dR_k}{dr}E = 0,\\
& -\frac{\omega_k H}{\sqrt{f}} + i\sqrt{f}\frac{dR_k}{dr}F = 0.
\end{aligned} \tag{34}$$

If there are non-trivial solutions in the above equations, the coefficient determinant must vanish. Therefore, we can get

$$\hbar^0 \quad -\frac{\omega_0^2}{f} + f\left(\frac{dR_0}{dr}\right)^2 + \frac{\lambda_q^2}{r^2} + \mu^2 = 0, \quad q = 1, 2, \tag{35}$$

$$\hbar^1 \quad -\frac{\omega_1^2}{f} + f\left(\frac{dR_1}{dr} + \frac{if'}{4f} + \frac{i}{r}\right)^2 = 0, \tag{36}$$

$$\hbar^k \quad k \ge 2 \quad -\frac{\omega_k^2}{f} + f\left(\frac{dR_k}{dr}\right)^2 = 0. \tag{37}$$

It is clear that (35) is none other than the radial semiclassical Hamilton-Jacobi equation. Solving equations (35)–(37), and then the incoming and outgoing probabilities of the fermion can be given by

$$\hbar^0: \quad P_0^{in} = \left|\Psi_0^{in}\right|^2 \sim e^{\frac{2}{\hbar}(\omega_0 \mathrm{Im}\, t + \mathrm{Im}\int \frac{\sqrt{\omega_0^2 - f(\lambda_0^2 r^{-2}+\mu^2)}}{f} dr)},$$
$$P_0^{out} = \left|\Psi_0^{out}\right|^2 \sim e^{\frac{2}{\hbar}(\omega_0 \mathrm{Im}\, t - \mathrm{Im}\int \frac{\sqrt{\omega_0^2 - f(\lambda_0^2 r^{-2}+\mu^2)}}{f} dr)}, \tag{38}$$

$$\hbar^1: \quad P_1^{in} = \left|\Psi_1^{in}\right|^2 \sim e^{\frac{2}{\hbar}(\omega_1 \mathrm{Im}\, t + \omega_1 \mathrm{Im}\int \frac{dr}{f} + \mathrm{Im}\int (\frac{f'}{4f} + \frac{1}{r}) dr)},$$
$$P_1^{out} = \left|\Psi_1^{out}\right|^2 \sim e^{\frac{2}{\hbar}(\omega_1 \mathrm{Im}\, t - \omega_1 \mathrm{Im}\int \frac{dr}{f} + \mathrm{Im}\int (\frac{f'}{4f} + \frac{1}{r}) dr)}, \tag{39}$$

$$\hbar^k: \quad k \geq 2 \; P_k^{in} = \left|\Psi_k^{in}\right|^2 \sim e^{\frac{2}{\hbar}(\omega_k \mathrm{Im}\, t + \omega_k \mathrm{Im}\int \frac{dr}{f})},$$
$$P_k^{out} = \left|\Psi_k^{out}\right|^2 \sim e^{\frac{2}{\hbar}\left(\omega_k \mathrm{Im}\, t - \omega_k \mathrm{Im}\int \frac{dr}{f}\right)}, \tag{40}$$

Now the incoming probability should be unified, so the outgoing probabilities are

$$\hbar^0: \quad P_0^{out} = \left|\Psi_0^{out}\right|^2 \sim e^{-\frac{4\pi\omega_0}{\hbar f'}}, \tag{41}$$

$$\hbar^1: \quad P_1^{out} = \left|\Psi_1^{out}\right|^2 \sim e^{-\frac{4\pi\omega_1}{\hbar f'}}, \tag{42}$$

$$\hbar^k: \quad k \geq 2 \quad P_k^{out} = \left|\Psi_k^{out}\right|^2 \sim e^{-\frac{4\pi\omega_k}{\hbar f'}}. \tag{43}$$

The total actions forms are the same, so the solutions of (41)–(43) are not independent, and each energy is proportional to semiclassical energy ω_0. In units $G = c = k_B = 1$, we therefore rewrite the radial action as

$$\omega = \omega_0 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^i}{S_{BH}^i} \omega_0 = \left(1 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^i}{S_{BH}^i}\right)\omega_0, \tag{44}$$

where S_{BH} is semiclassical Bekenstein Hawking entropy. The correctional total fermion tunneling outgoing probability is obtained

$$P^{out} \sim \exp\left[-\left(1 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^i}{S_{BH}^i}\right)\frac{4\pi\omega_0}{\hbar f'}\right]. \tag{45}$$

Via the principle of "detailed balance"

$$P^{out} \sim e^{-\frac{\omega_0}{T_h}} P^{in} = e^{-\frac{\omega_0}{T_h}}, \tag{46}$$

the correctional Hawking temperature can be given by

$$T_h = \left(1 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^i}{S_{BH}^i}\right)^{-1} T_H. \tag{47}$$

where T_H is semiclassical Hawking temperature, which is

$$T_H = \frac{\hbar}{4\pi} f'(r_0). \tag{48}$$

Next, let's research the correctional entropy. In black hole thermodynamics, the famous law states

$$dM = T_h dS_{bh} + \Theta dQ + \Omega dJ, \tag{49}$$

where Θ, Q, J and Ω are black holes' electromagnetic potential, electric charge, angular momentum and angular velocity respectively. It is clear that the correctional entropy of static cylindrical symmetric black hole is $dS_{bh} = \frac{dM}{T_h}$, so we can obtain

$$S_{bh} = \int dS_{bh} = \int \frac{dM}{T_h} = S_{BH} + 4\pi\beta_1 \ln S_{BH} + const + \cdots. \tag{50}$$

Our conclusion support Majhi's work [55]. In this paper, we have never left out any trivial terms, so our work shows that semiclassical entropy of cylindrical symmetric black hole should be corrected. Because actual black holes are stationary or dynamical, and some resent work proves several higher-dimensional black holes are stable [66–69], so it is worthwhile to research these cases. Therefore, what we will do is research quantum tunneling from stationary, dynamical and higher-dimensional black holes via the method beyond semiclassical approximation in the future.

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